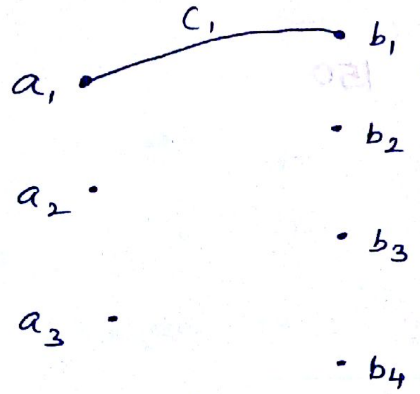
UNIT - II

Transportation Model

* Transportation deals with transportation of a commodity (single product) from 'm' sources (supply) to 'n' destinations (demand).



x_{ij} → Quantity transported from i to j

Minimize $Z = \sum \sum C_{ij} x_{ij}$

$\sum x_{ij} \leq a_i$, $i = 1, 2, 3, \ldots$

$\sum x_{ij} \geq b_j$, $j = 1, 2, 3, \ldots$

$x_{ij} \geq 0$

$\boxed{a_i}$ Supply $\boxed{C_{ij}}$ Transport cost $\boxed{b_j}$ Demand

* $\boxed{\sum a_i \geq \sum b_j}$

Types!

① Balanced → $\sum a_i = \sum b_j$

② Unbalanced → $\sum a_i \neq \sum b_j$

Methods:

Step ①: Basic feasible Solution

1) North west corner rule (NWC)

2) Least cost method (LCM)

3) Vogel's approximation method (VAM)

Step ②: Optimal Solution

1) Stepping Stone method

2) Modified Distribution method (MODI)

① Find the basic feasible solution for the following problem using ① NWC ② Least Cost ③ VAM

					Supply
	4	6	8	8	40
	6	8	6	7	60
	5	7	6	8	50
Demand	20	30	50	50	150

Solution:

① North West Corner rule:

$\boxed{n + m - 1 \text{ allocations}}$ → Maximum allocations

n → Supply

m → Demand

NWC

4 20	6 20	8	8	~~40~~ ~~20~~
6	8 10	6 50	7	~~60~~ ~~50~~
5	7	6	8 50	~~50~~
~~20~~	~~30~~ ~~10~~	~~50~~	~~50~~	

$$\therefore \text{cost} = (4 \times 20) + (6 \times 20) + (8 \times 10) + (6 \times 50) + (8 \times 50)$$

$\boxed{\text{Total cost} = 980}$ → Basic feasible Solution

③ Least Cost method (or) Minimum cost method:

Least cost

1, 2, 3, 4, 5, 6 → order

(I)

4 (1) 20	6 (2) 20	8	8	~~40~~ ~~20~~
6	8	6 (3) 50	7 (4) 10	60 ~~10~~
5	7 (5) 10	6	8 (6) 40	50 40
~~20~~	30 ~~10~~	~~50~~	50 40	

$\therefore$ Transport cost $= (4\times20)+(6\times20)+(6\times50)+(7\times10)$
$+(7\times10)+(8\times40)$

$\Rightarrow$ $\boxed{\text{Transport cost} = \text{Rs. } 960}$

(II)

(T)

4 (1) 20	6 (3) 20	8	8	~~40~~ ~~20~~
6	8	6 (2) 50	7 (5) 10	~~60~~ ~~10~~
5	7 (4) 10	6	8 (6) 40	50 ~~40~~
~~20~~	~~30~~ ~~10~~	~~50~~	~~50~~ ~~40~~	

$\boxed{\text{Transport cost} = \text{Rs. } 960/-}$

(III)

(4) 1 20	(6) 3 20	8	8	~~40~~ ~~20~~
6	(8) 5 10	6	(7) 4 50	~~60~~ ~~10~~
5	7	(6) 2 50	8	~~50~~
~~20~~	~~30~~ ~~10~~	~~50~~	~~50~~	

$\therefore$ Cost $= (4\times20)+(6\times20)+(8\times10)+(7\times50)+(6\times50)$

$\Rightarrow$ $\boxed{\text{Transport Cost} = \text{Rs. } 930/-}$

(3) Vogel's Approximation method (VAM):

High Penalty $\rightarrow$ Try to put as much as least as

(I)

					1	2			
4 20	6 20	8	8	~~40~~ ~~20~~	(2) 6-4	(2)			
6	8	6 10	7 50	~~60~~ ~~50~~	(0) 6-6	(1)	(1)	(1)	(1) 5
5	7 10	6 40	8	~~50~~ ~~40~~	(1) 6-5	(1)	(1)	(2) 4	
~~20~~	~~30~~ ~~10~~	~~50~~ ~~10~~	~~50~~						
(1) 5-4	(1) 7-6	(0) 6-6	(1) 8-7						
	(1)	(0)	(1)						
	3 (1)	(0)	(1)						
		(0)	(1)						

$\therefore$ cost $= (4\times20)+(6\times20)+(6\times10)+(7\times50)+(7\times10)+(6\times40)$

$\rightarrow$ $\boxed{\text{Transport cost} = \text{Rs. } 920/-}$

(II)

4 — 20	6 — 20	8	8	~~20~~ ~~40~~
6	8	6 — 50	7 — 10	~~10~~ ~~60~~
5	7 — 10	6	8 — 40	~~40~~ ~~50~~
~~20~~	~~30~~ ~~10~~	~~50~~	~~50~~ ~~40~~	

$\therefore$ Cost $= (4\times20)+(6\times20)+(6\times50)+(7\times10)+(7\times10)+(8\times40)$

$\Rightarrow$ $\boxed{\text{Transport cost} = \text{Rs. } 960/-}$

Conditions:

1. IBFS should not having a loop.
2. Independent allocations must be equal to $m+n-1$
3. If degenerative Solution occurs then the extra allocation must be marked as ε.
   $\hookrightarrow$ $< m+n-1 \Rightarrow$ Add ε
4. If ε is going to be add then ensure that it should not forms a loop.
5. Do not more than $m+n-1$.
   $\hookrightarrow$ should having a loop. (Avoid this)

Loop formation & Breaking:

①

4 10 (-1)	6	8	8 30 (+1)	40
6	8	6 50	7 10	60
5 10 (+1)	7 30	6	8 10 (-1)	50
20	30	50	50	

X

$\therefore$ Change in cost $= -4 + 5 - 8 + 8 = 1$ ⇒ Not desirable

Final table:

4	6	8	8 40	40
6	8	6 50	7 10	60
5 20	7 30	8	8	50
20	30	50	50	

②

4 10 (+1)	6	8	8 30 (-1)	40
6	8	6 50	7 10	60
5 10 (-1)	7 30	6	8 10 (+1)	50
20	30	50	50	

✓

$\therefore$ Change in cost $= 4 - 5 + 8 - 8 = -1$ ⇒ Desirable

Final table:

4 20	6	8	8 20	40
6	8	6 50	7 10	60
5	7 30	6	8 20	50
20	30	50	50	

Optimal Solution : [MODI (or) U-V method]

* Basic Solution is obtained by using LCM (or) VAM

① $C_{ij} = u_i + v_j$

② $d_{ij} = C_{ij} - (u_i + v_j)$

Iteration - 1:

C_{ij}

	$v_1 = 4$	$v_2 = 6$	$v_3 = 6$	$v_4 = 7$	
$u_1 = 0$	4 20	6 20	8 (2)	8 (1)	40
$u_2 = 0$	6 (2)	8 (2)	6 50 $-\theta$	7 10 $+\theta$	60
$u_3 = 1$	5 (0)	7 10	6 $+\theta$ (-1)	8 40 $-\theta$	50
	20	30	50	50	

From this minimum value is taken as θ

$\boxed{\theta = 40}$

Iteration - 2:

	$V_1 = 4$	$V_2 = 6$	$V_3 = 5$	$V_4 = 6$	
$u_1 = 0$	4 / 20	6 / 20	8 / (3)	8 / (2)	40
$u_2 = 1$	6 / (1)	8 / (1)	6 / 10	7 / 50	60
$u_3 = 1$	5 / (0)	7 / 10	6 / 40	8 / (1)	50
	20	30	50	50	

'0' indicates alternate optimum

Since $\boxed{\text{All } d_{ij} \geq 0}$

$\therefore$ The Solution reaches optimal.

$\therefore$ The cost $= (4\times20) + (6\times20) + (6\times10) + (7\times50) + (7\times10) + (6\times40)$

$\Rightarrow$ $\boxed{\text{Transport cost} = \text{Rs. } 920/-}$

Allocation Table:

4 / 20	6 / 20		
		6 / 10	7 / 50
	7 / 10	6 / 40	

Alternate Solution:

	$V_1=4$	$V_2=6$	$V_3=5$	$V_4=6$	
$u_1=0$	4 / 20 $-\theta$	6 / 20 $+\theta$	8 / (3)	8 / (2)	40
$u_2=1$	6 / (1)	8 / (1)	6 / 10	7 / 50	60
$u_3=1$	5 / $+\theta$ (0)	7 / 10 $-\theta$	6 / 40	8 / (1)	50
	20	30	50	50	

$\boxed{\theta = 10}$

Final Table:

4 / 10	6 / 30		
		6 / 10	7 / 50
5 / 10		6 / 40	

For Degenerate Solution – Optimal :

From NWC answer ↳ $\nearrow$ $< m+n-1$

	$V_1 = 4$	$V_2 = 6$	$V_3 = 4$	$V_4 = 6$	
$u_1 = 0$	4 / 20	6 / 20	8 / (4)	8 / (2)	40
$u_2 = 2$	6 / (0)	8 / 10 $-\theta$	6 / 50 $+\theta$	7 / (−1)	60
$u_3 = 2$	5 / (−1)	7 / $+\theta$ (−1)	6 / $-\theta$ ε	8 / 50	50
	20	30	50	50	

$\boxed{\theta = \varepsilon}$

Iteration-2 :

	$V_1 = 4$	$V_2 = 6$	$V_3 = 4$	$V_4 = 7$	
$u_1 = 0$	4 / 20	6 / 20	8 / (4)	8 / (1)	40
$u_2 = 2$	6 / (0)	8 / $-\theta$ $10-\varepsilon$	6 / $50+\varepsilon$	7 / $+\theta$ (−2)	60
$u_3 = 1$	5 / (0)	7 / $+\theta$ ε	6 / (1)	8 / $-\theta$ 50	50
	20	30	50	50	

$\boxed{\theta = 10-\varepsilon}$

Iteration-3 :

	$V_1 = 4$	$V_2 = 6$	$V_3 = 6$	$V_4 = 7$	
$u_1 = 0$	4 / 20	6 / 20	8 / (2)	8 / (1)	40
$u_2 = 0$	6 / (2)	8 / (2)	6 / $50+\varepsilon$ $-\theta$	7 / $10-\varepsilon$ $+\theta$	60
$u_3 = 1$	5 / (0)	7 / 10	6 / $+\theta$ (−1)	8 / $-\theta$ $40+\varepsilon$	50
	20	30	50	50	

$\boxed{\theta = 40+\varepsilon}$

Iteration-4:

	$v_1 = 4$	$v_2 = 6$	$v_3 = 5$	$v_4 = 6$	
$u_1 = 0$	4 / 20	6 / 20	8 / (3)	8 / (2)	40
$u_2 = 1$	6 / (1)	8 / (1)	6 / 10	7 / 50	60
$u_3 = 1$	5 / (0)	7 / 10	6 / $40+\varepsilon$	8 / (1)	50
	20	30	50	50	

Since all $d_{ij} \geq 0$.

$\therefore$ The Solution reaches optimal.

The Transport cost $= 4 \times 20 + 6 \times 20 + 6 \times 10 + 7 \times 50$
$+ 60 \times (40+\varepsilon) + 7 \times 10$

$\Rightarrow$ $\boxed{\text{Transport cost} = \text{Rs. } 920/-}$

$\varepsilon \to 0$

Unbalanced and Maximization T.P:

* For unbalanced $\Sigma a_i \neq \Sigma b_j$
  - Add a dummy row (or) dummy column.

* For maximization
  - Subtract all the entries by highest entry.
  (or)
  - Multiply by -1 with all the entries.

1) Solve the following T.P for maximization of Profit.

Profit / units

	A	B	C	D	Supply
	40	25	22	33	100
	44	35	30	30	30
	38	38	28	30	70
Demand	40	20	60	30	

Solution:

The given problem is maximization of profit.

$\therefore$ Highest entry $= 44$. Subtract all entries by 44

4	19	22	11	100
0	9	14	14	30
6	6	16	14	70
40	20	60	30	

Now the total supply $\Sigma b_i = 100+30+70 = 200$

Total Demand $\Sigma a_i = 40+20+60+30 = 150$

$\Sigma a_i \neq \Sigma b_i \Rightarrow$ Unbalanced T.P.

For balancing of T.P add a dummy column.

4	19	22	11	0	100
0	9	14	14	0	30
6	6	16	14	0	70
40	20	60	30	50	

Iteration - 1: (LCM)

4 10	19 ×	22 10	11 30	0 50	~~100~~ ~~50~~ ~~40~~ 10
0 30	9 ×	14 ×	14 ×	0	~~30~~
8 ×	6 20	16 50	14 ×	0	~~70~~ ~~50~~
~~10~~ ~~40~~	~~20~~	~~60~~ ~~10~~	~~30~~	~~50~~	

$X_{11} = 10$; $X_{13} = 10$; $X_{14} = 30$; $X_{15} = 50$

$X_{21} = 30$; $X_{32} = 20$; $X_{33} = 50$

$\therefore$ Transport cost $= 10 \times 40 + 10 \times 22 + 30 \times 33 + 50 \times 0 +$

$30 \times 44 + 20 \times 38 + 50 \times 28$

$$\boxed{\text{Profit} = \text{Rs. } 5090/-}$$

Iteration - 2: (MODI)

	$V_1 = 4$	$V_2 = 12$	$V_3 = 22$	$V_4 = 11$	$V_5 = 0$	
$u_1 = 0$	4 +θ 10	19 (7)	22 −θ 10	11 30	0 50	100
$u_2 = -4$	0 30 −θ	9 (1)	14 +θ (-4)	14 (7)	0 (4)	30
$u_3 = -6$	6 (8)	6 20	16 50	14 (9)	0 (6)	70
	40	20	60	30	50	

$$\boxed{\theta = 10}$$

Iteration-3:

	$v_1=4$	$v_2=8$	$v_3=18$	$v_4=11$	$v_5=0$	
$u_1=0$	4 20	19 (11)	22 (4)	11 30	0 50	100
$u_2=-4$	0 20	9 (5)	14 10	14 (7)	0 (4)	30
$u_3=-2$	6 (4)	6 20	16 50	14 (5)	0 (2)	70
	40	20	60	30	50	

Since All $\boxed{d_{ij} \geq 0}$. The Solution reaches optimal.

$\therefore$ Allocation is

$X_{11} = 20$ $\quad X_{23} = 10$

$X_{14} = 30$ $\quad X_{32} = 20$

$X_{15} = 50$ $\quad X_{33} = 50$

$X_{21} = 20$

$\therefore$ Profit $= 20 \times 40 + 30 \times 33 + 50 \times 0 + 20 \times 44 + 10 \times 30 + 20 \times 38 + 50 \times 28$

$\boxed{\text{Profit} = \text{Rs. } 5130/-}$

ASSIGNMENT MODEL

* It is a particular case of transportation problem.
* Objective is to assign number of tasks.
* m x n matrix → cost matrix (or) Effectiveness matrix

Hungarian method:

① Subtract the smallest entry of each row from all entries of each row. (Row minimum)

② Subtract the smallest entry of each column from all entries of each column. (Column minimum)

③ Examine the rows successively until one exact zero is found and cross all the other zeros in the column. (Assign one zero in each row)

④ Examine the columns successively until one exact zero is found and cross all the other zeros in the row. (Assign one zero in each column)

⑤ If each row and each column contains exactly one zero then the solution is optimal. (Exact zeros optimal)

⑥ If each row and each column does not contain one zero then the solution is not optimal.

(Does not zero is not optimal)

⑥ If solution is not optimal then do this things.

- Mark the rows that do not have assignment.
- Mark the columns that have zeros in marked rows.
- Mark the rows that have assignment in columns.
- Draw lines through all unmarked rows and marked columns.

Find the smallest element(s) is not covered.
Subtract(s) all elements not covered.
Single line crossing do not change values.
Double line passes then add (s).

① Solve the following assignment problem:

$$\begin{array}{c|c} & \text{Job} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Person} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix} \end{array}$$

Solution:

Cost matrix = 5×5 ⇒ Balanced

Iteration-1:

	1	2	3	4	5
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Iteration - 2:

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline A & 7 & 3 & 0 & 5 & 0 \\ B & 0 & 9 & 4 & 5 & 4 \\ C & 1 & 6 & 6 & 0 & 4 \\ D & 4 & 3 & 0 & 0 & 3 \\ E & 4 & 0 & 2 & 4 & 0 \\ \hline \end{array}$$

Iteration - 3:

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline A & 7 & 3 & 0 & 5 & \boxed{0} \\ B & \boxed{0} & 9 & 4 & 5 & 4 \\ C & 1 & 6 & 6 & \boxed{0} & 4 \\ D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ E & 4 & \boxed{0} & 2 & 4 & \cancel{0} \\ \hline \end{array}$$

Since each row and each column ~~has~~ contains exactly one zero (assignment). The solution is optimal.

∴ The optimum assignment is: A → 5, D → 3, B → 1, E → 2, C → 4

∴ Assignment cost = 1 + 0 + 2 + 1 + 5

⇒ $\boxed{\text{Assignment cost} = \text{Rs.}9/-}$

(2) The Processing time in hours for the jobs when allocated to the different machines are indicated below. Assign the machines for the jobs so that total processing time is minimum.

Machines

Jobs	M_1	M_2	M_3	M_4	M_5
J_1	9	22	58	11	19
J_2	43	78	72	50	63
J_3	41	28	91	37	45
J_4	74	42	27	49	39
J_5	36	11	57	22	25

Solution:

The cost matrix =
$$\begin{pmatrix} 9 & 22 & 58 & 11 & 19 \\ 43 & 78 & 72 & 50 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix}_{5\times5} \Rightarrow \text{Balanced}$$

Iteration - 1:

	M_1	M_2	M_3	M_4	M_5
J_1	0	13	49	2	10
J_2	0	35	29	7	20
J_3	13	0	63	9	17
J_4	47	15	0	22	12
J_5	25	0	46	11	14

Iteration - 2 :

$$\begin{array}{c|ccccc|}
 & M_1 & M_2 & M_3 & M_4 & M_5 \\
\hline
J_1 & 0 & 13 & 49 & 0 & 0 \\
J_2 & 0 & 35 & 29 & 5 & 10 \\
J_3 & 13 & 0 & 63 & 7 & 7 \\
J_4 & 47 & 15 & 0 & 20 & 2 \\
J_5 & 25 & 0 & 46 & 9 & 4 \\
\hline
\end{array}$$

Iteration - 3 :

$$\begin{array}{c|ccccc|c}
 & M_1 & M_2 & M_3 & M_4 & M_5 & \\
\hline
J_1 & \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} & \\
J_2 & \boxed{0} & 35 & 29 & 5 & 10 & \\
J_3 & 13 & \boxed{0} & 63 & 7 & 7 & \checkmark \\
J_4 & 47 & 15 & \boxed{0} & 20 & 2 & \\
J_5 & 25 & \cancel{0} & 46 & 9 & 4 & \checkmark \\
\hline
 & & \checkmark & & & & \\
\end{array}$$

$\boxed{\theta = 4}$

Iteration - 4 :

$$\begin{array}{c|ccccc|}
 & M_1 & M_2 & M_3 & M_4 & M_5 \\
\hline
J_1 & \cancel{0} & 17 & 49 & \boxed{0} & \cancel{0} \\
J_2 & \boxed{0} & 39 & 29 & 5 & 10 \\
J_3 & 9 & \boxed{0} & 59 & 3 & 3 \\
J_4 & 47 & 19 & \boxed{0} & 20 & 2 \\
J_5 & 21 & \cancel{0} & 42 & 5 & \boxed{0} \\
\hline
\end{array}$$

Since each row and each column contains exactly one assignment. ∴ The solution reaches optimal.

$J_1 \to M_4$

$J_2 \to M_1$

$J_3 \to M_2$

$J_4 \to M_3$

$J_5 \to M_5$

∴ Processing time = $11 + 43 + 28 + 27 + 25$

$\Rightarrow$ $\boxed{\text{Processing time} = 134 \text{ hrs}}$

Unbalanced Assignment Problem:

① A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

Machines

		1	2	3	4
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

What are the assignments which will minimize the cost?

Solution:

The cost matrix $= 3 \times 4 \Rightarrow$ Unbalanced

$\therefore$ A dummy row is added.

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{4 \times 4}$$

Iteration-1:

	1	2	3	4
A	0	6	10	14
B	0	5	9	11
C	0	5	9	12
D	0	0	0	0

Iteration-2:

	1	2	3	4	
A	$\boxed{0}$	6	10	14	✓
B	$\cancel{0}$	5	9	11	✓
C	$\cancel{0}$	5	9	12	✓
D	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
	✓				

$\boxed{\theta = 5}$

Iteration-3:

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline A & \boxed{0} & 1 & 5 & 9 \\ B & \cancel{0} & \boxed{0} & 4 & 6 \\ C & \cancel{0} & \cancel{0} & 4 & 7 \\ D & 5 & \cancel{0} & \boxed{0} & \cancel{0} \\ \hline \end{array}$$

$$\boxed{\theta = 4}$$

Iteration-4:

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline A & \boxed{0} & 1 & 1 & 5 \\ B & \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ C & \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ D & 9 & 4 & \cancel{0} & \boxed{0} \\ \hline \end{array}$$

Since each row and each column contains one assignment

∴ The solution reaches optimal.

Optimal Solution:	Alternate Solution:
A → 1	A → 1
B → 2	B → 3
C → 3	C → 2
D → 4	D → 4

∴ The assignment cost = 18 + 13 + 19 + 0 = Rs. 50/-

⇒ $\boxed{\text{Assignment cost} = \text{Rs. } 50/-}$

Maximization Case in A.P:

① Solve the assignment problem for maximization given the profit matrix (in rupees).

Machines

Jobs	P	Q	R	S
A	51	53	54	50
B	47	50	48	50
C	49	50	60	61
D	63	64	60	60

Solution:

The cost matrix = 4×4 ⇒ Balanced

Given problem is maximization of profit.

∴ Subtract all values from 64.

	P	Q	R	S
A	13	11	10	14
B	17	14	16	14
C	15	14	4	3
D	1	0	4	4

Iteration - 1:

	P	Q	R	S
A	3	1	0	4
B	3	0	2	0
C	12	11	1	0
D	1	0	4	4

Iteration- 2:

	P	Q	R	S
A	2	1	$\boxed{0}$	4
B	2	$\boxed{0}$	2	$\cancel{0}$
C	11	11	1	$\boxed{0}$
D	$\boxed{0}$	$\cancel{0}$	4	4

Since each row and each column contains one assignment.

∴ The Solution reaches optimal.

A → R

B → Q

C → S

D → P

∴ Assignment cost = 54 + 50 + 61 + 63

⇒ Assignment Profit = Rs. 228/-

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② Solve the following assignment of Salesman to maximize the annual sales. (Amount in thousands of Rs.)

Territories

Salesman	1	2	3	4
1	60	50	40	30
2	40	30	20	15
3	40	20	35	10
4	30	30	25	20

Solution:

The cost matrix = 4×4 ⇒ Balanced.

Given problem is maximization of Profit.

∴ Subtract all values with 60.

$$
\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 10 & 20 & 30 \\
2 & 20 & 30 & 40 & 45 \\
3 & 20 & 40 & 25 & 50 \\
4 & 30 & 30 & 35 & 40 \\
\hline
\end{array}
$$

Iteration - 1:

$$
\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 10 & 20 & 30 \\
2 & 0 & 10 & 20 & 25 \\
3 & 0 & 20 & 5 & 30 \\
4 & 0 & 0 & 5 & 10 \\
\hline
\end{array}
$$

Iteration - 2:

$$
\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & \boxed{0} & 10 & 15 & 20 \\
2 & \cancel{0} & 10 & 15 & 15 \\
3 & \cancel{0} & 20 & \boxed{0} & 20 \\
4 & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \\
\hline
\end{array}
$$

$\boxed{\theta = 10}$

Iteration - 3:

$$
\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & \boxed{0} & \cancel{0} & 5 & 10 \\
2 & \cancel{0} & \boxed{0} & 5 & 5 \\
3 & 10 & 20 & \boxed{0} & 20 \\
4 & 10 & \cancel{0} & \cancel{0} & \boxed{0} \\
\hline
\end{array}
$$

Since each row & column contains one assignment.

$\therefore$ The solution reaches optimal.

Salesman 1 → Territory 1	**Alternate:** S1 → T2
Salesman 2 → Territory 2	S2 → T1
Salesman 3 → Territory 3	S3 → T3
Salesman 4 → Territory 4	S4 → T4

$\therefore$ Profit = 60 + 30 + 35 + 20

⇒ $\boxed{\text{Profit} = \text{Rs. } 145/- \text{ (in thousands)}}$

Travelling Salesman Problem (TSP):

* A Salesman must visit a no. of cities starting from his head quarters.
* Objective → Find shortest distance (or min. time (or) min. cost)
* Restriction → In diagonal only.

① Solve the following TSP.

To

From	A	B	C	D
A	–	46	16	40
B	41	–	50	40
C	82	32	–	60
D	40	40	36	–

Solution:

Cost matrix = 4×4 ⇒ Balanced

Iteration-1:

$$
\begin{array}{c|cccc|}
 & A & B & C & D \\
\hline
A & \infty & 30 & 0 & 24 \\
B & 1 & \infty & 10 & 0 \\
C & 50 & 0 & \infty & 28 \\
D & 4 & 4 & 0 & \infty \\
\hline
\end{array}
$$

Iteration-2:

$$
\begin{array}{c|cccc|}
 & A & B & C & D \\
\hline
A & \infty & 30 & \boxed{0} & 24 \\
B & \boxed{0} & \infty & 10 & \cancel{0} \\
C & 49 & \boxed{0} & \infty & 28 \\
D & 3 & 4 & \cancel{0} & \infty \\
\hline
\end{array}
$$

$\boxed{\theta = 3}$

Iteration-3:

$$
\begin{array}{c|cccc|}
 & A & B & C & D \\
\hline
A & \infty & 27 & \boxed{0} & 21 \\
B & \cancel{0} & \infty & 13 & \boxed{0} \\
C & 49 & \boxed{0} & \infty & 28 \\
D & \boxed{0} & 1 & \cancel{0} & \infty \\
\hline
\end{array}
$$

Since each row & column contains exactly one assignment.

∴ The solution reaches optimal.

The optimal assignment schedule is,

A → C

B → D

C → B

D → A

i.e, $\boxed{A \rightarrow C \rightarrow B \rightarrow D \rightarrow A}$

minimum cost = 16 + 40 + 32 + 40

⇒ $\boxed{\text{min. cost} = \text{Rs. } 128/-}$